

# Anonymous Credentials

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\* part of this set is based on J. Camenisch slides (IBM Zurich)



# In this Session

- Anonymous credentials

- Building Blocks
- **WHAT** are they?
- **WHY** do we need them?
- **HOW** to build them?

- Source\*:

Jan Camenisch, Gregory Neven and Anja Lehmann  
IBM Research - Zurich

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\* the slides are not the original: they were redesigned for this presentation



# Cryptographic Building Blocks

- Blinding Signatures
- Zero-knowledge proofs
- Commitments

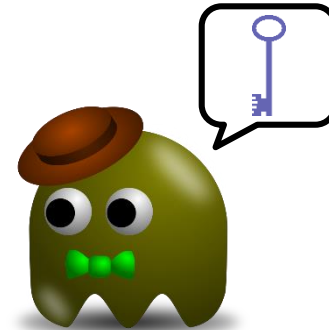
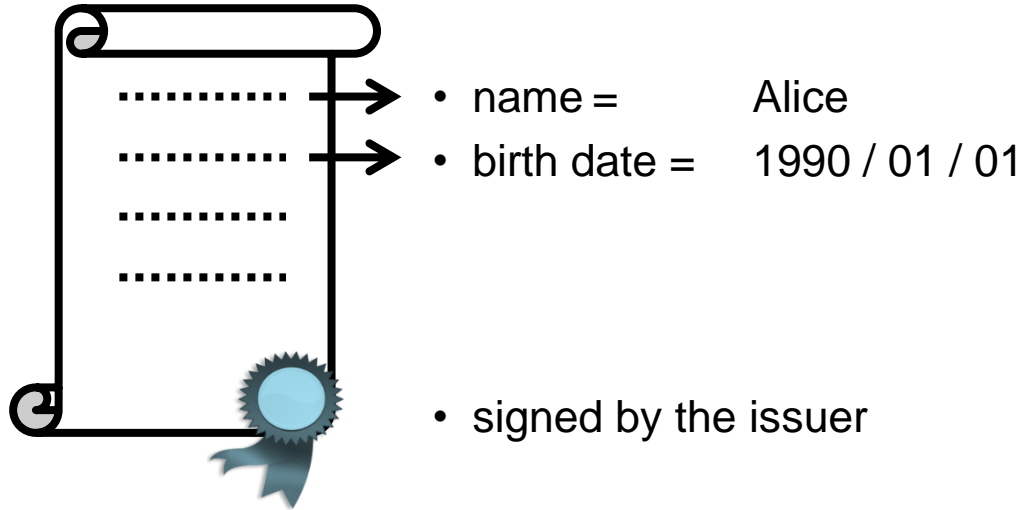


# Today

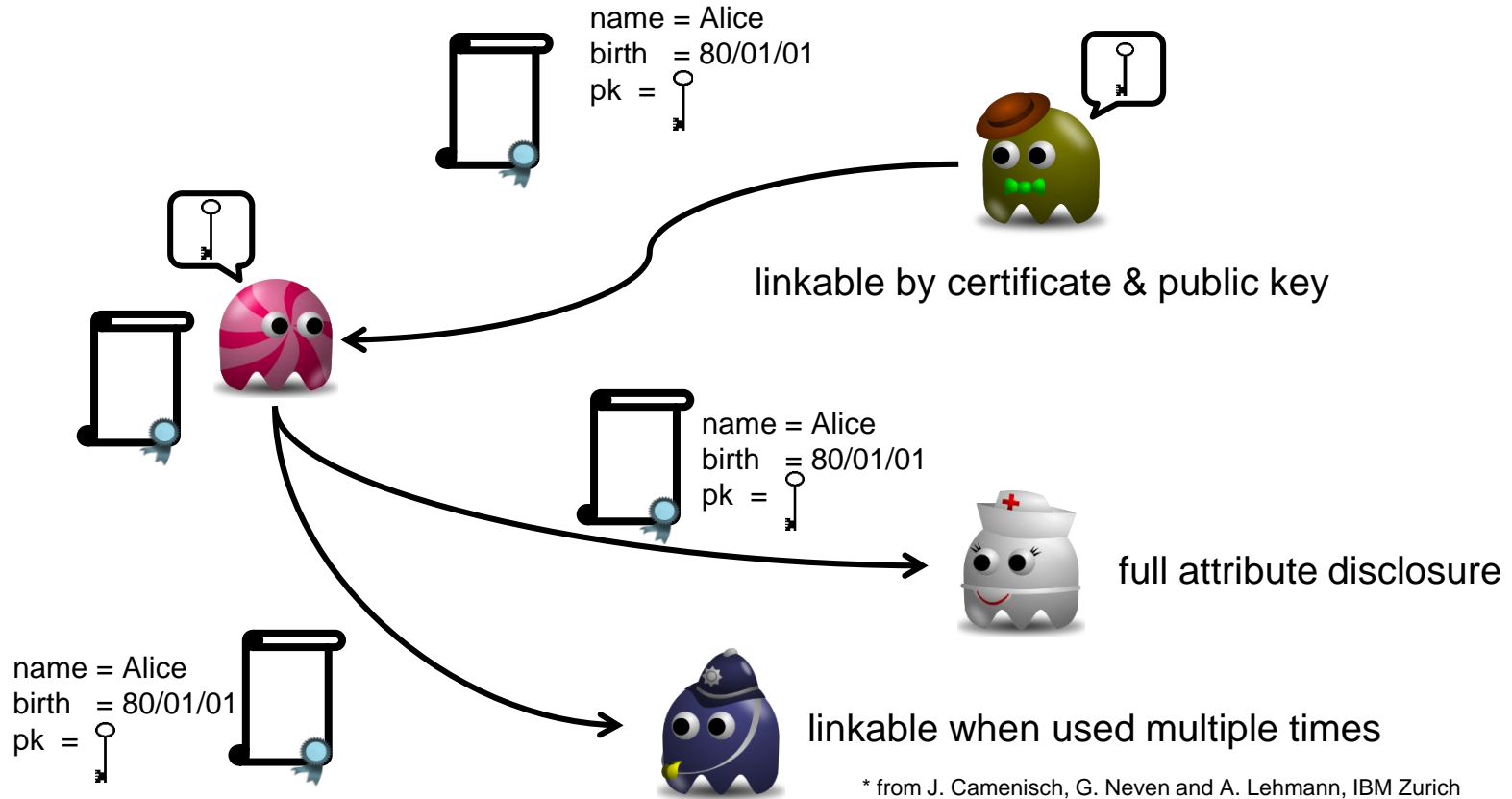
- Credentials and anonymous credentials
  - **WHAT** are they?
  - **WHY** do we need them?
  - **HOW** to build them?
- Their building blocks
- How can you use them

# Credentials and Certificates

- A signed list of attribute-value pairs



# Problems with X.509 Public-Key Certificates



\* from J. Camenisch, G. Neven and A. Lehmann, IBM Zurich

# Anonymous Credentials

- **ARE** privacy-enhancing attribute based credentials  
(privacy-ABCs)

≈ minimal disclosure token



a sort of identification  
(similar to a certificate)

- There are different ways to build them  
e.g. Identify Mixer, U-Prove



# Anonymous Credentials: Basic Functionality

- Protects the users privacy
  - Anonymity
  - Unlinkability (multi-use)
  - Selective disclosure



evil siblings



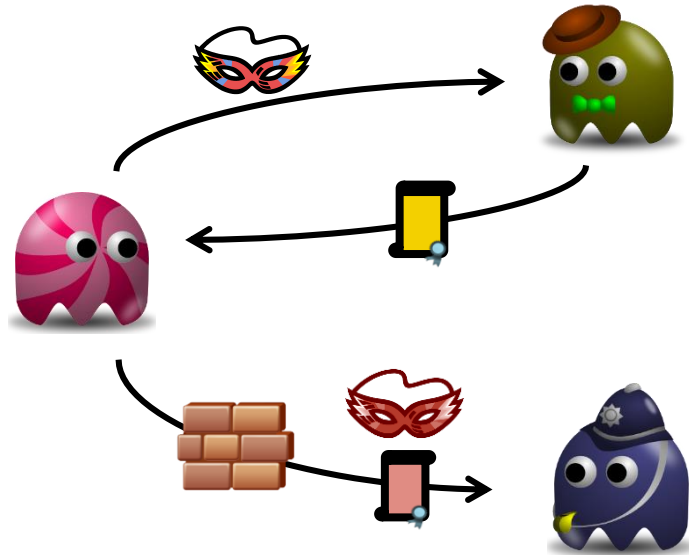
evil Alice

Unforgeability of credentials  
Consistency of credentials (no sharing)

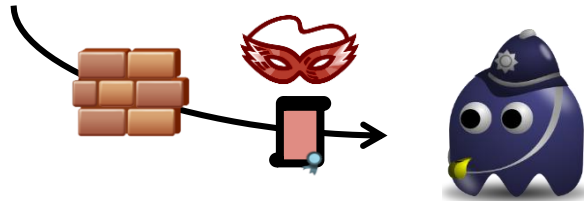


# The two-ways to build them

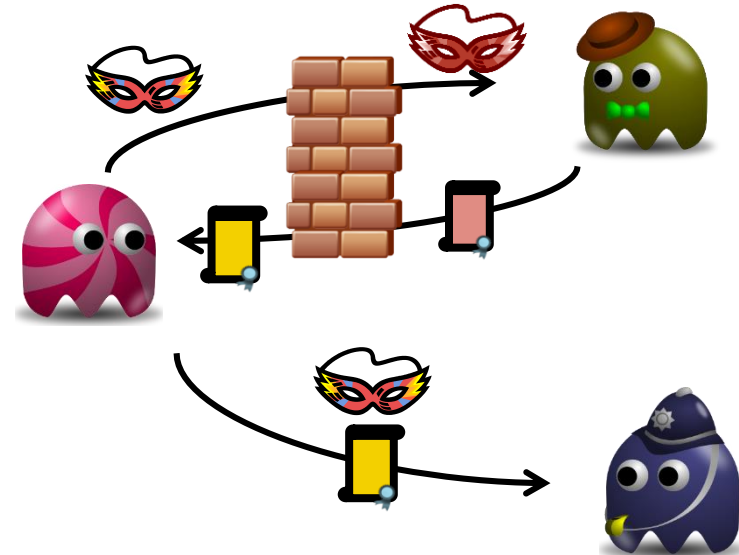
## Zero-Knowledge Proofs



## Identity Mixer



## Blind Signatures



## U-Prove

\* from J. Camenisch, G. Neven and A. Lehmann, IBM Zurich

# Blind Signatures and Zero-Knowledge Proofs

- Blind Signatures
  - The problem:  
How can a Signer sign something without seeing it ?
- Zero-Knowledge Proofs
  - The problem:  
How to prove that I know the answer to a problem  
without telling the answer ?



# Blind Signatures

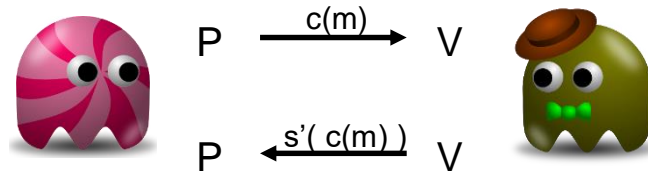
- Enables a signer to sign a message where the content has been blinded (i.e. hidden) prior to the signature
- A verifier that is:
  - *The signer* can verify the signature  
**BUT**  
**not link it to the message** it signed earlier
  - *A third party* can also verify the signature



# Blind Signatures

- Given the functions

- $s(x)$  (public) and (the inverse of  $s$ )  $s'(x)$  (secret)      Signer V
- $c(x)$  (public) and (the inverse of  $c$ )  $c'(x)$  (secret)      Prover P



$$c'(s'(c(m))) = s'(m)$$



Blinded Signature!

# Blind Signatures: RSA

- signature  $s = m^d \bmod n$
- public key  $(e, n)$  and  $n = p \cdot q$  (large primes)
- private key  $(d, n)$

- P calculates  $m' = m \cdot r^e \bmod n$   
↓  
blinding factor

$$\varphi(n) = (p - 1)(q - 1)$$
$$d = e^{-1} \bmod \varphi(n)$$

$$P \xrightarrow{m'} V$$

$$P \xleftarrow{s'} V \quad \text{where } s' = (m')^d \bmod n$$

$$s' = (m \cdot r^e)^d \bmod n = m^d r^{e \cdot d} \bmod n = m^d r \bmod n$$

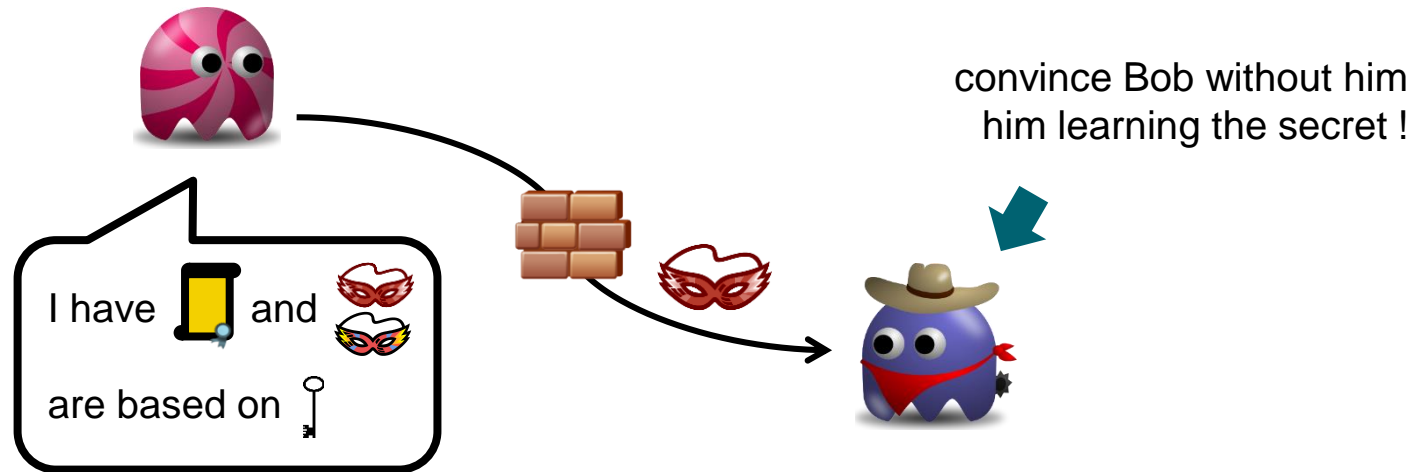
$$s = s' \cdot r^{-1} = m^d \bmod n$$

➡ Blinded Signature



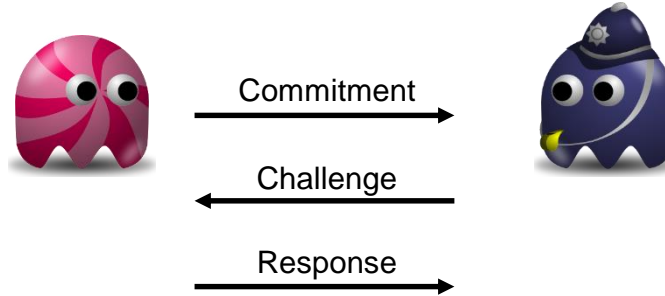
# Zero-Knowledge Proofs

- A provider wants to convince a verifier that she knows a secret *without* revealing the secret!



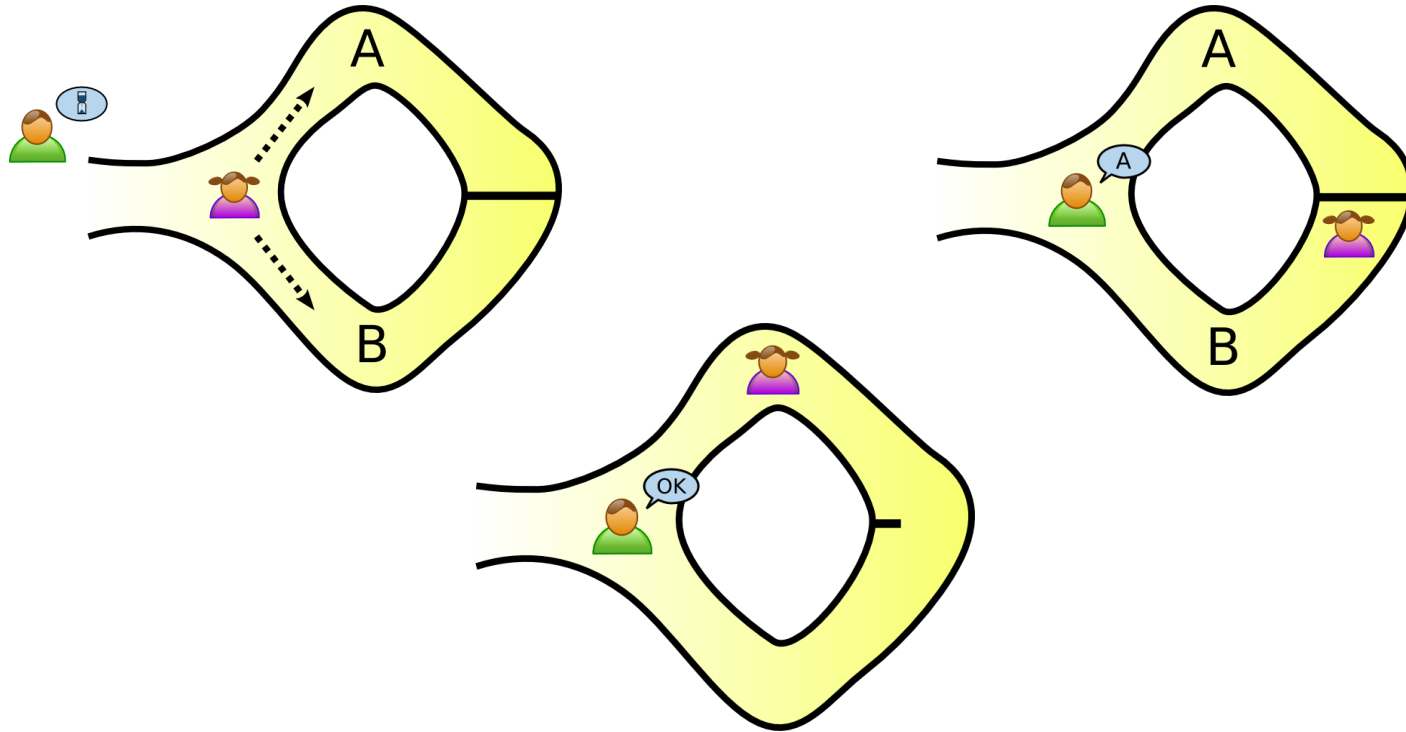
# Zero-Knowledge Proofs

- Interactive proof



- Properties:
  - Zero-knowledge      verifier learns nothing about the prover's secret
  - Soundness            prover can convince verifier only if she knows the secret
  - Completeness        if prover knows the secret she can always convince the verifier

# Zero-Knowledge Proofs in Pictures



\* Quisquater, 1990





# Zero-Knowledge Proofs with Discrete Logs

- Given group  $G$  and element  $y \in G$ 
  - prove knowledge of  $x = \log_g y$  such that verifier only learns  $g$  and  $y$



$(x, g, y)$   
Prover

$$PK\{ (x): y = g^x \}$$

$(g, y)$   
Verifier



random  $r$

$$t := g^r$$

$t$

$c$

random  $c$

$$s := r - x \cdot c$$

$s$

accept if  $t = g^s y^c$

Proof of completeness:

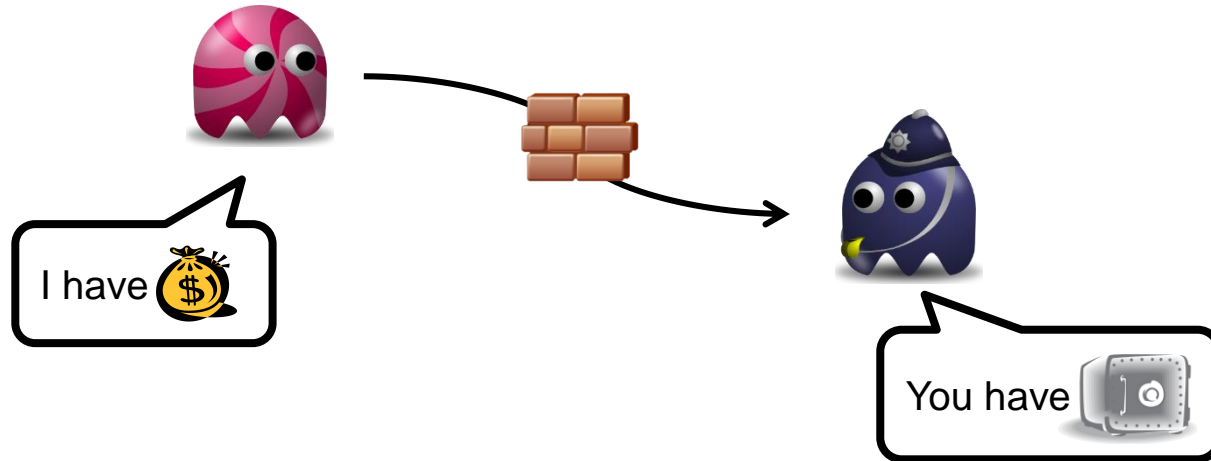
$$g^s y^c = g^{r-x \cdot c} y^c = g^{r-x \cdot c} g^{x \cdot c} = g^r = t$$

\* Schnorr, 1991



# Commitment Schemes

- The problem:  
How to prove that you hold (or selected) a value, and  
commit to it (so it can be proved later), without showing it ?



# Commitment Schemes

- Allows a sender to commit to a value (the commitment) towards a receiver, without revealing the value
  - the sender can later reveal (open) the value to the receiver

$$c \leftarrow \text{Commit}(m)$$

$$\{true, false\} \leftarrow \text{Open}(c, m)$$

- The commitment has to be binding
- The analogy: coin flipping over the telephone




# Commitment Schemes

## A coin flipping over the telephone

- Bob flips the coin and tells the result to Alice
  - we use bits instead

 Not fair!

- Alice picks a random bit  $a$  and sends it to Bob and Bob picks a random bit  $b$  and sends it to Alice
  - the value of the coin is  $= a \oplus b$

 Who goes first?

- Alice commits to her bit  $c \leftarrow \text{Commit}(a)$   
Bob sends  $b$  to Alice  
Alice sends  $a$  to Bob and Bob opens the commitment  
 $\{\text{true}, \text{false}\} \leftarrow \text{Open}(c, a)$



# Pedersen Commitment Scheme

- A scheme that offers

- unconditional hiding
- computational binding



does not leak information about  $m$

given  $c, m, r$  it is hard to compute  $m' \neq m$  and  $r' \neq r$  such that

$$true \leftarrow Open(c, m', r')$$

- homomorphic function

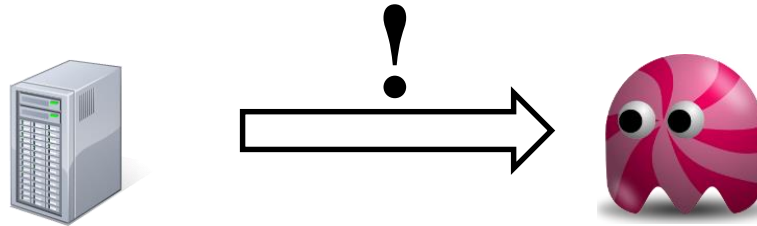
$$Commit(a, r) \cdot Commit(b, s) = Commit(a + b, r + s)$$

- To commit  $m$ , pick random  $r$  and compute  $c = g^m h^r$   
and for opening the commitment, reveal  $(m, r)$

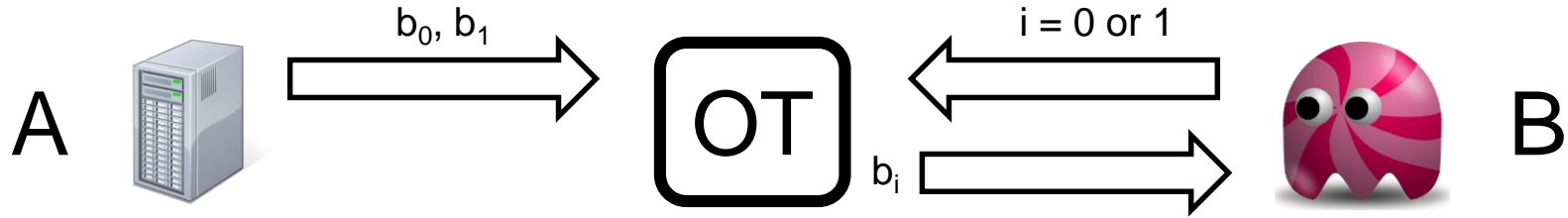


# Oblivious Transfer

- The problem:  
How to transfer information to a receiver,  
and not know what information was transferred?  
  
(i.e. to protect the receiver's privacy)



# Oblivious Transfer: the basic idea



- A inputs 2 bits and B inputs the index of one of A's bits
- B learns his chosen bit, A learns nothing
  - A does not learn which bit B has chosen
  - B does not learn the value of the bit that he did not choose
- Generalizes to bitstrings (n instead of 2)

\* from on Vitaly Shmatikov



# Oblivious Transfer

## the 1-2 OT protocol

A 2 messages  $m_0, m_1$   
 $x_0, x_1$  ( $d, e, N$ )



B

random  $x_0, x_1$

$x_0, x_1$

picks  $b$  ( $x_0$  or  $x_1$ ),  
random  $k$ , blinds  $x_b$   
 $v = (x_b + k^e)$

$v$

doesn't know  $b$   
 $k_0 = (v - x_0)^d$  and  
 $k_1 = (v - x_1)^d$ .

$m'_0 = m_0 + k_0$

$m'_1 = m_1 + k_1$

$m_b = m'_b - k$

A does not know which message B could unblind



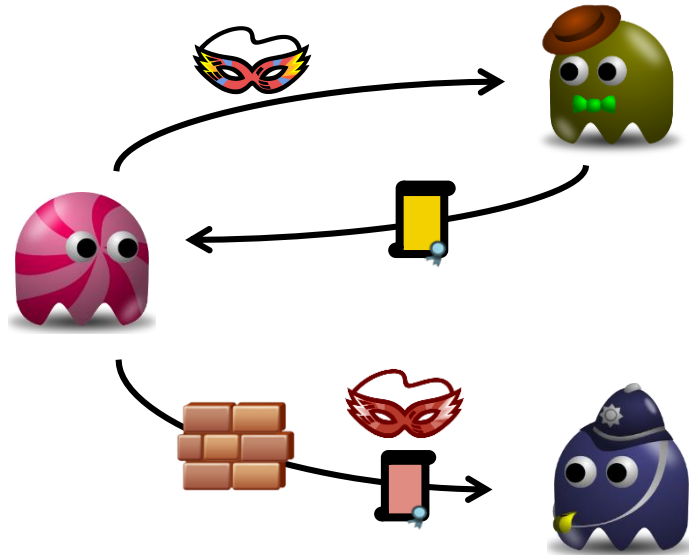


# and now back to Anonymous Credentials

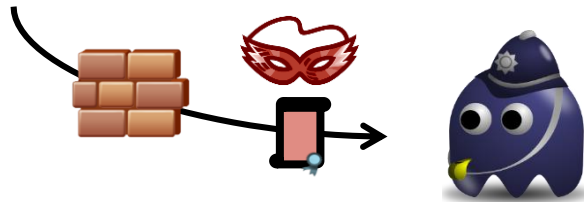


# The two-ways to build them

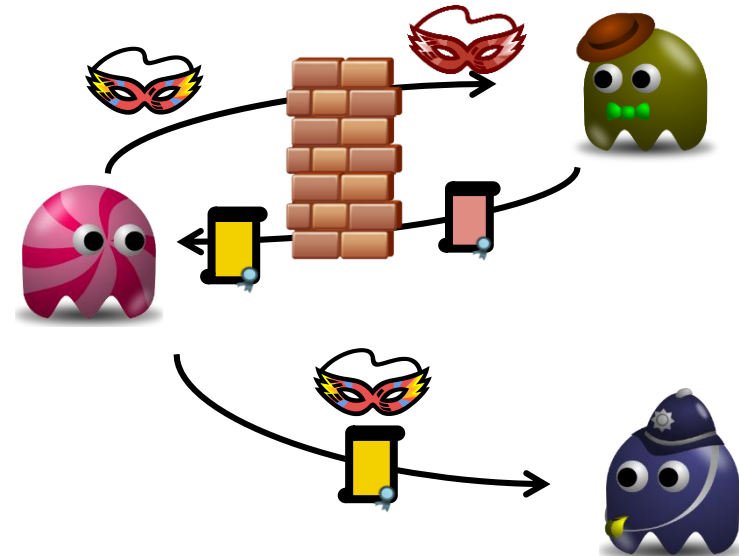
## Zero-Knowledge Proofs



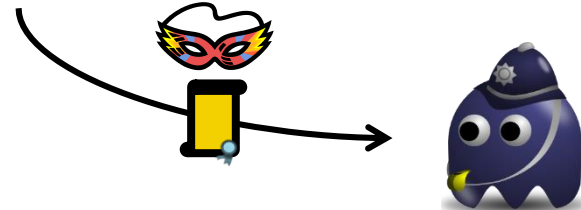
## Identity Mixer



## Blind Signatures



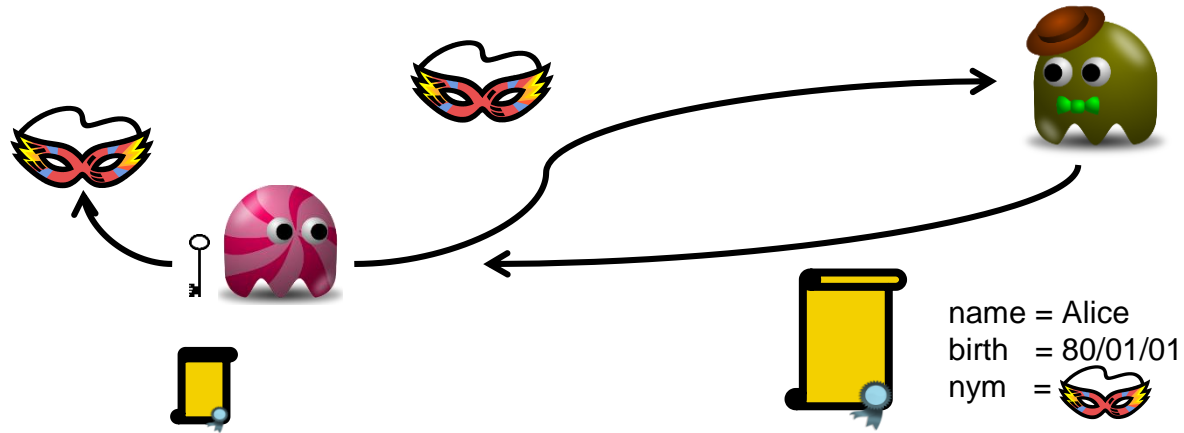
## U-Prove



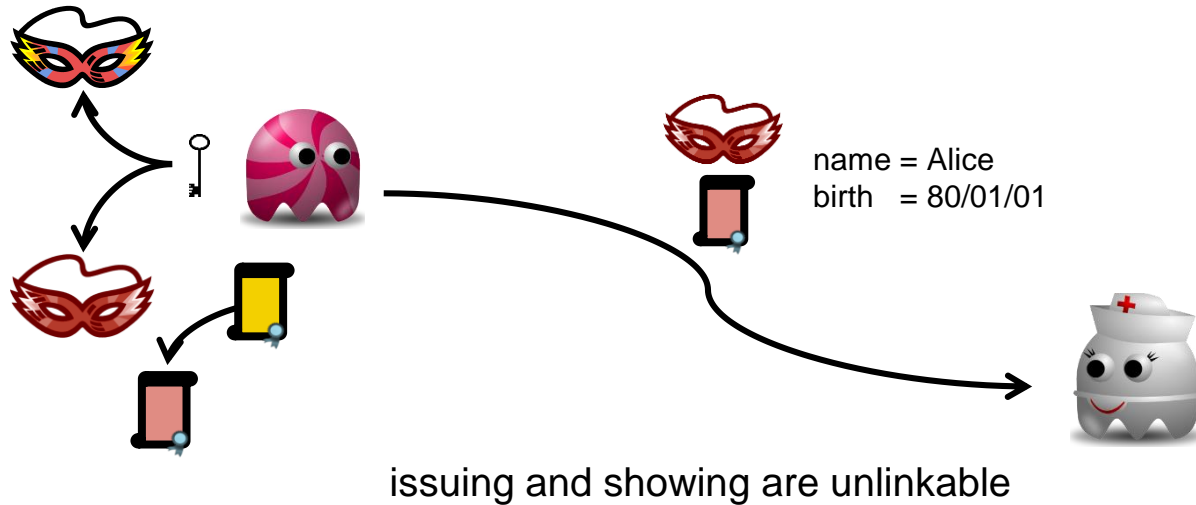
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# Anonymous Credentials Generation

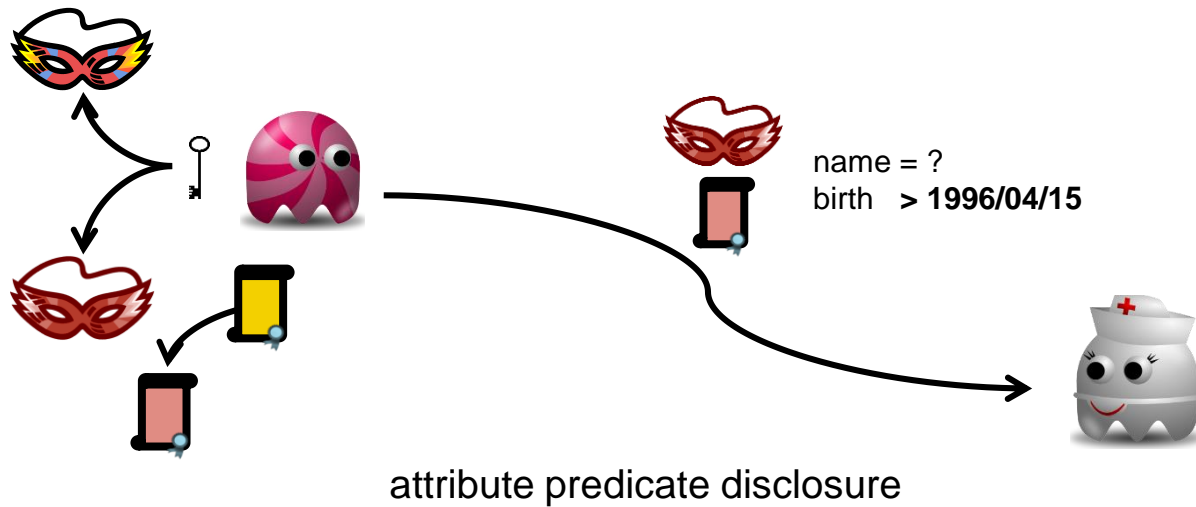


# Anonymous Credential Using them

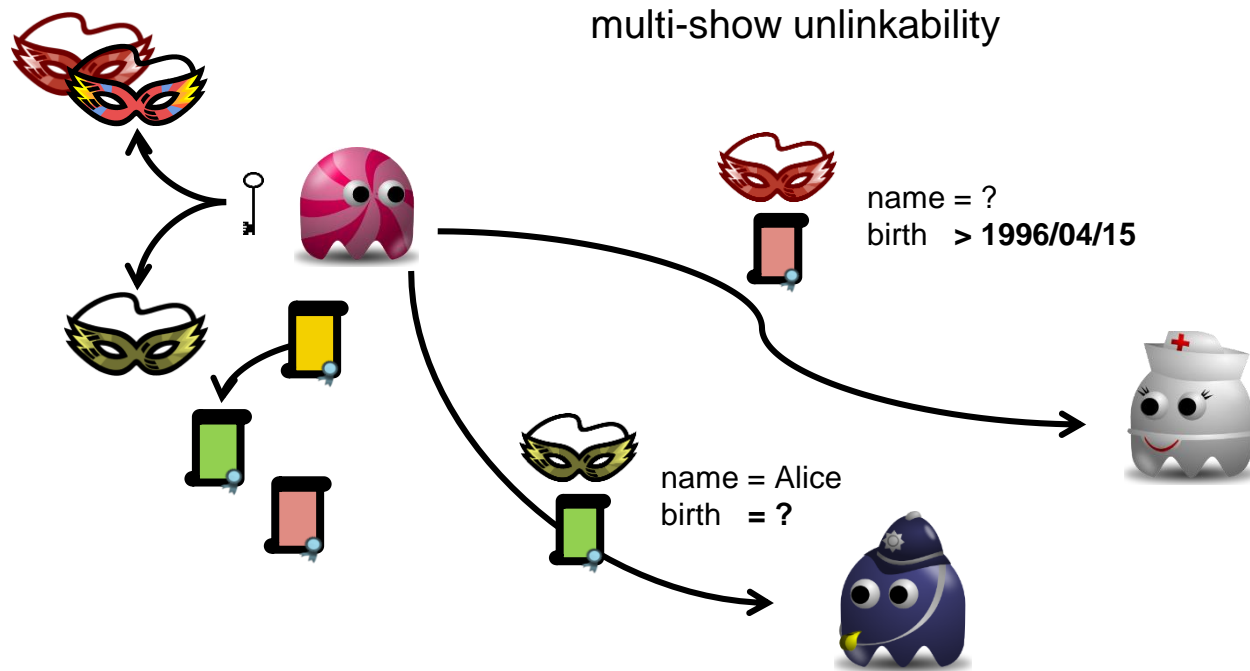




# Anonymous Credential Using them

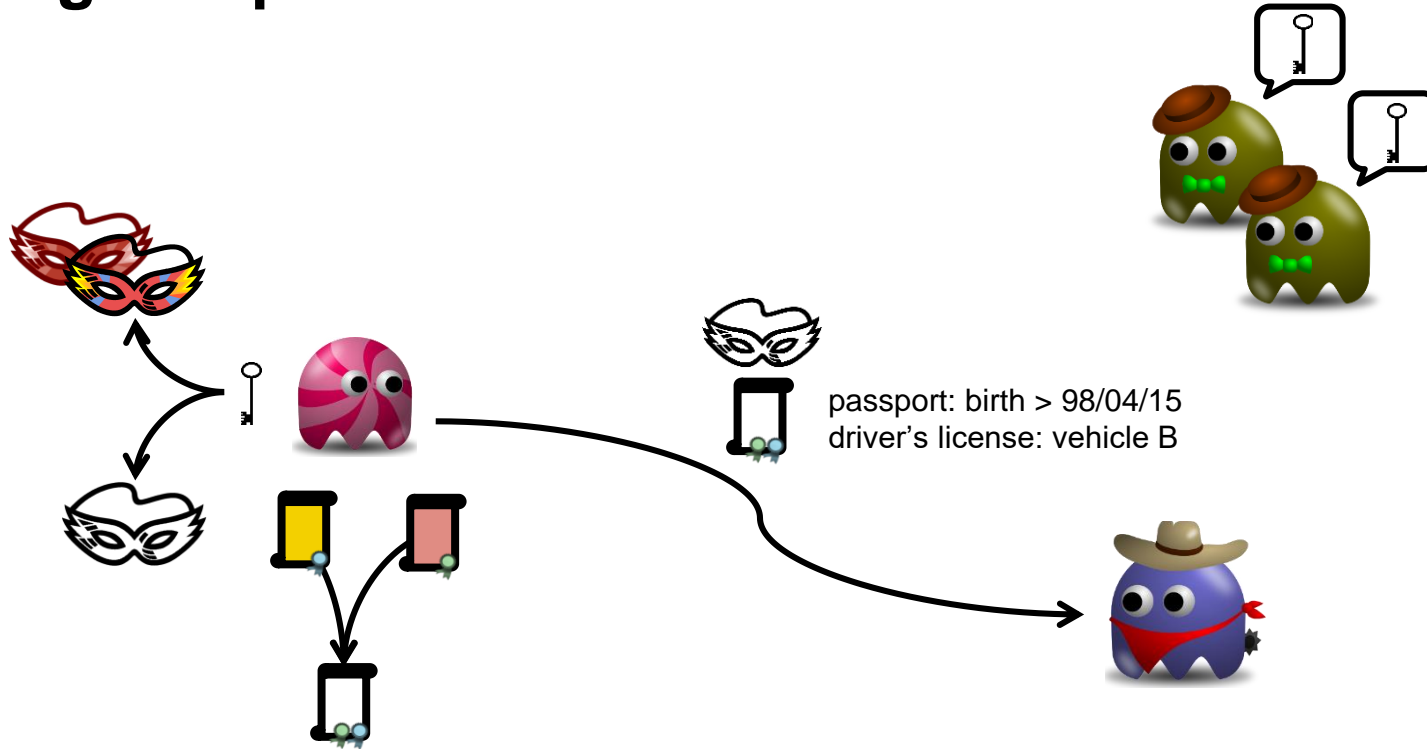


# Anonymous Credential Using them



# Anonymous Credential

## Using multiple of them





# Summary

- Anonymous credentials enable data minimization
  - Only reveal the attributes needed or just prove some predicate over them
- Strong yet privacy preserving authentication
- Identity Mixer and U-Prove are similar
  - implementations are available
  - various extensions are possible



# Questions?

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# Extended Functionalities of Anonymous Credentials

- Credentials on hidden attributes
- Tracing of user/attribute
- Revocation of credentials
  - Accumulators, signed intervals, validity time
- Limited spending
  - Hidden serial number, domain pseudonyms, offline identity recovery, verifiable random functions

